

Spherically symmetric inhomogeneous model with Chaplygin gas

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Abstract

We investigate the late time acceleration with a Chaplygin type of gas in spherically symmetric inhomogeneous model. At the early phase we get Einstein-deSitter type of solution generalised to inhomogeneous spacetime. But at late stage of the evolution our solutions admit the accelerating nature of the universe. For a large scale factor our model behaves like a Λ CDM model. We calculate the deceleration parameter for this anisotropic model, which, unlike its homogeneous counterpart, shows that the *flip* is not synchronous occurring early at the outer shells. This is in line with other physical processes in any inhomogeneous models. Depending upon initial conditions our solution also gives bouncing universe. In the absence of inhomogeneity our solution reduces to wellknown solutions in homogeneous case. We have also calculated the effective deceleration parameter in terms of Hubble parameter. The whole situation is later discussed with the help of wellknown Raychaudhuri equation and the results are compared with the previous case. This work is an extension of our recent communication where an attempt was made to see if the presence of extra dimensions and/or inhomogeneity can trigger an inflation in a matter dominated Lemaitre Tolman Bondi model.

KEYWORDS : cosmology; accelerating universe; inhomogeneity;
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1 Introduction

Following the high redshift supernovae data in the last decade [1] we know that when interpreted within the framework of the standard FRW type of universe (homogeneous and isotropic) we are left with the only alternative that the universe is now going through an accelerated expansion with baryonic matter contributing only five percent of the total budget. Later data from CMBR studies [2] further corroborate this conclusion which has led a vast chunk of cosmology community ([3] and references therein) to embark on a quest to explain the cause of the acceleration. The teething problem now confronting researchers is the identification of the mechanism that triggered the late inflation. Workers in this field are broadly divided

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into two groups - either modification of the original general theory of relativity or introduction of any mysterious fluid in the form of an evolving cosmological constant or a quintessential type of scalar field. But as discussed extensively in the literature (we are sparing the readers here to repeat once again those arguments) both the alternatives face serious theoretical problems. In this context one important thing should not escape our attention. One intriguing fact in the framework of the standard FRW model is that the accelerating phase coincides with the period in which inhomogeneities in the matter distribution at length scales < 10 Mpc become significant so that the Universe can no longer be approximated as homogeneous at these scales. One should also note that homogeneity and isotropy of the geometry are not essential ingredients to establish a number of relevant results in relativistic cosmology. One need not be too sacrosanct about these concepts so as to sacrifice basic physics (energy conditions, for example) in relativistic cosmology. Conversely, if the universe is not *a priori* assumed to be homogeneous and isotropic, the observational data do not necessarily imply an accelerating expansion of the universe, or even if the cosmic expansion is accelerating it does not necessarily point to an existence of a dark energy. Thus to account for the observational data without introducing the concept of dark energy, varied arguments regarding the effects of *inhomogeneities* have been made and naturally a vast community of cosmologists have embarked upon a sort of ‘mission’ to explain (sometimes with conflicting claims) the observational findings within inhomogeneous models. The immediate generalisation of FRW spacetime is the wellknown LTB model [4] which is also spherically symmetric but the spacetime is inhomogeneous. However, the assumption of spherical symmetry requires a centre of the universe so that the observer be located not too far from the centre to avoid undetected large anisotropy (a detailed study of LTB and allied cosmologies and its relevance to current astrophysical issues may be found in [5]). The *sojourn* to the inhomogeneous path has a chequered history. *Naively* speaking there are two such arguments. One is that the apparent acceleration of the cosmology can be regarded as a result of an almost spherically symmetric but inhomogeneous peculiar velocity field, assuming that we are located at the vicinity of the symmetry centre [6, 7]. With this argument the acceleration of the cosmic volume expansion is not necessary. The other argument is that the acceleration of the universe is a physical reality and results from the backreaction effects due to the inhomogeneities in the background FRW universe [8, 9]. This idea is later supplemented by Carter *et al* [10] where the observed universe is assumed to be an underdense bubble in an Einstein-de Sitter universe and it was shown that from observational point of view their results become very similar to the predictions of Λ CDM model. However, in a recent communication Bolejko and Andersson [11] have calculated the volume deceleration based on Buchert averaging scheme and back reaction in some LTB models and have shown that for realistic cases the deceleration parameter turns out to be positive. At this stage a very brief mention of the formalism may not be out of place. The difference between the evolution of homogeneous models and an inhomogeneous universe is caused by backreaction effects, due to non linearity of Einstein equations such that the solutions for a homogeneous matter distribution leads in principle to a different description of the universe than an average of an inhomogeneous solution to the exact Einstein equations. So either we have to fall upon on exact solutions or invoke averaging the backreaction terms. If simply a vol-

ume averaging is considered then such an attempt leads to Buchert equation. The Buchert equations are very similar to Friedmann equations except for the backreaction term, which is, in general nonvanishing if inhomogeneity is present (for a lucid review of the averaging scheme the reader is referred to [12, 13]). Moreover, the validity of the perturbative ansatz is questionable in that the claimed acceleration is later shown to be due to the result of extrapolation of a specific solution to a regime where both the perturbative expansion breaks down and the constraints are violated [14]. On the otherhand Kai *et al* [15] showed that if one hypothesizes the coexistence of expanding and contracting regions in space, the speed of the cosmic volume expansion can be accelerated. These models are constructed by replacing the spherical domains from the Einstein-deSitter model with a LTB dust sphere having the same gravitational mass. Another interesting suggestion has recently come from the works of Wiltshire *et al* [16] where a timescape cosmology has been proposed as a viable alternative to homogeneous cosmologies with dark energy. It realises cosmic acceleration as an apparent effect that arises in calibrating average cosmological parameters in the presence of spatial curvature and gravitational energy gradients that grow large with the growth of inhomogeneities at late epochs. The model is based on an exact solution to a Buchert average of the Einstein equations with backreaction. Some people attempted to look into the problem from a purely geometric point of view - an approach more in line with Einstein's spirits. For example, Panigrahi *et al* [17] have recently toyed with the idea of dimension driven acceleration in a number of publications, where the extra terms coming from the higher dimensions create a sort of back reaction to drive inflation. Good thing about it is that one need not have to invent any exotic, unphysical matter field in this case. In an interesting contribution Wanas [18] introduced torsion to explain late acceleration. It is shown that torsion generates a new type of energy to be called torsion energy which is repulsive in nature, thus mimicing a quintessential type of field. While torsion inspired inflation has several desirable features (for example, geometrical origin) the problem with Wanas' model is that the geometry no longer remains Riemannian.

Again we know [19] that in a matter dominated nonrotating model where particles interacting with one another move along geodesic lines it is always possible to define a coordinate system which is at once synchronous ($g_{00} = 1$) and comoving. With this input Hirata and Seljak [20] claimed to have proved from Raychaudhuri equation [21] that in a perfect fluid cosmological model that is geodesic, rotation-free and obeys the strong energy condition ($\rho + 3p \geq 0$), a certain generalisation of the deceleration parameter, q_4 must be always non-negative. But even with the perturbation considered by Kolb *et al* [8] the vorticity vanishes and consequently Kolb's claim is flawed. On the other hand, Iguchi *et al* [6] did obtain simulated acceleration in Lemaitre-Tolman (LT) models with $\Lambda = 0$ that obey the conditions set by Hirata *et al*, which subsequently led Vanderveld *et al* [7] to draw attention to this apparent contradiction between these two conclusions and to suggest that LT models that simulate accelerated expansion also contain a weak singularity, and in that case the derivation of HS breaks down. In addition to this, there are other singularities that tend to arise in LT models, and Vanderveld *et al* [7] have failed to find any singularity-free models that agree with observations. However in a pioneering contribution Krasinski *et al* [22] neatly summed up the apparent controversies

as also the claims and counter claims of different workers in this field and showed that the so called weak singularity is not a singularity at all, while the other types of singularities like shell crossing or shell focussing, generic to all inhomogeneous collapse may be taken care of with suitably chosen arbitrary functions appearing in the theory. Moreover, one should point out at this stage that unlike the homogeneous case it is always difficult, if not a little ambiguous to define uniquely a deceleration parameter for inhomogeneous models. In this context Krasinski *et al* [22] also showed that Hirata-Seljak's formulation of q_4 is wrong, based on inadmissible averaging of the redshifts over directions and with the averaging dropped one gets correct signature of the deceleration parameter which may be both positive or negative. On the other hand Hansson *et al* [23] argued that when taking the real, inhomogeneous and anisotropic matter distribution in the semi-local universe into account, there may be no need to postulate an accelerating expansion of the universe at all despite recent type Ia supernova data. In fact inhomogeneous structure formation may alleviate need for accelerating universe.

In the present work our goal is completely different. In a recent communication one of us [24] examined the possibility if the presence of inhomogeneity or extra dimensions separately or jointly in an LTB model can achieve late acceleration without the aid of any extraneous scalar field. We found that while dimensions have no perceptible effect on nature of evolution the radial or angular acceleration is possible even in pure dust distribution if any of them decelerates fast enough in LTB model. The present work is an extension over that in the sense that we here introduce a Chaplygin gas as input. The lack of information regarding the provenance of dark matter and dark energy allows for speculation with economical and aesthetic idea that a single component acted in fact as both dark matter and dark energy. The unification of these two components has risen considerable theoretical interest, because on the one hand the model building has become considerably simpler and on the other hand such unification implies the existence of an era during which the energy densities of dark matter and dark energy are remarkably similar. Moreover, at present it is unclear whether the backreaction effects of inhomogeneities can actually accelerate the cosmic volume expansion.

One possible way to achieve that unification is through a particular k-essence fluid, the Chaplygin gas with the exotic equation of state. While literature abounds with work on Chaplygin gas in FRW models ([25] and references therein), barring a few [26] we are not aware of works of similar kind directed to inhomogeneous space-time. However, relevant to mention that Gorini *et al* [27] have recently discussed a Tolman-Oppenheimer-Volkoff type of bounded distribution in presence of a generalised Chaplygin gas. Here we discuss the evolution of a spherically symmetric inhomogeneous model with a Chaplygin type of matter field and get the interesting result that an initially decelerating phase transits to a late accelerating one in line with the current observational results. We compared our findings with those obtained via Raychaudhuri equation also and get identical results. Organisation of our paper is as follows. In section 2 we solve the field equations for our inhomogeneous spacetime with Chaplygin gas as matter field and have addressed the problems for both early and late time inflation assuming a flat 3 space under different subtitles. While our solutions are amenable to both early deceleration and late

acceleration we get an interesting result that under suitable initial conditions the model also admits a bouncing type of universe avoiding the big crunch. In line with inhomogeneous collapse the bounce occurs at different instants for different shells unlike the synchronous homogeneous case. In section 3 we compared our findings with the conclusions coming also from the Raychaudhuri equation. The paper ends with some concluding remarks in section 4.

2 Field Equation

$$ds^2 = dt^2 - X^2 dr^2 - R^2(r, t) dX^2 \quad (1)$$

where dX^2 represents a 2-sphere with

$$dX^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad (2)$$

and the scale factor, $R(r, t)$ depends both on space and radial coordinates (r, t) respectively. A prime overhead denotes $\partial/\partial r$ and a dot denotes $\partial/\partial t$. As pointed out in the last section the gauge, $g_{00} = 1$ follows for a spherically symmetric, irrotational inhomogeneous system only when, $p = 0$. But in our case, $p = 0$ only under extremal condition and not all through its evolution. So the *ansatz*, $g_{00} = 1$ should be treated as an additional assumption to simplify field equations [28].

A comoving coordinate system is taken such that $u^0 = 1, u^i = 0$ ($i = 0, 1, 2, 3$) and $g^{\mu\nu}u_\mu u_\nu = 1$ where u_i is the 4-velocity. The energy momentum tensor for a dust distribution in the above defined coordinates is given by

$$T^\mu_\nu = (\rho + p)\delta^\mu_0\delta^0_\nu - p\delta^\mu_\nu \quad (3)$$

where $\rho(r, t)$ is the matter density and $p(r, t)$ is the pressure. The fluid consists of successive shells marked by r , whose local density is time-dependent. The function $R(r, t)$ describes the location of the shell marked by r at the time t . Through an appropriate rescaling it can be chosen to satisfy the gauge

$$R(0, r) = r \quad (4)$$

The independent field equations for the metric (1) and the energy momentum tensor (3) are given by

$$G^0_0 = \frac{2\dot{X}\dot{R}}{XR} + \frac{1 + \dot{R}^2}{R^2} - \frac{1}{X^2} \left(\frac{2R''}{R} + \frac{R'^2}{R^2} - 2\frac{X'R'}{XR} \right) = \rho \quad (5)$$

$$G^1_1 = 2\frac{\ddot{R}}{R} + \frac{1 + \dot{R}^2}{R^2} - \frac{R'^2}{R^2 X^2} = -p \quad (6)$$

$$G^2_2 = G^3_3 = \frac{\ddot{X}}{X} + \frac{\ddot{R}}{R} + \frac{\dot{X}\dot{R}}{XR} - \frac{1}{X^2} \left(\frac{R''}{R} - \frac{X'R'}{XR} \right) = -p \quad (7)$$

$$G^0_1 = -2 \left(\frac{\dot{R}'}{R} - \frac{R'}{R} \frac{\dot{X}}{X} \right) = 0 \quad (8)$$

Solving $G_{01} = 0$ equation we get

$$X(r, t) = \frac{R'}{f(r)} \quad (9)$$

where $f(r)$ is an arbitrary function of r .

Since the WMAP data [13] shows that the universe is spatially flat to within a few percent we can take $f = 1$ such that the field equations finally reduce to the following two independent equations as

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} = \rho \quad (10)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = -p \quad (11)$$

From the the Bianchi identity we get for the inhomogeneous model the conservation law

$$\nabla_\nu T^{\mu\nu} = 0 \quad (12)$$

which, in turn, yields

$$\delta_\mu p + \frac{1}{\sqrt{-g}}\delta_\nu [\sqrt{-g}(\rho + p)u^\mu u^\nu] + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda = 0 \quad (13)$$

For $\Gamma_{00}^\mu = 0$ and $\sqrt{-g} = X(r, t)R^2(r, t)\sin\theta$, we obtain

$$\frac{d\rho}{dt} + \frac{1}{XR^2}\frac{d}{dt}(XR^2)(\rho + p) = 0 \quad (14)$$

At this stage we assume that we are dealing with a Chaplygin type of gas obeying an equation of state

$$p = -\frac{A}{\rho} \quad (15)$$

where A is a positive constant. It was first introduced as a cosmological fluid unifying dark matter and dark energy by Kamenshchik *et al* [25] and since has been widely studied in this context. Moreover it has found applications in particle physics via string theory [29] and its supersymmetric extension [30]. With this input we finally get

$$\dot{\rho} + \frac{1}{XR^2}\frac{d}{dt}(XR^2)\left(\rho - \frac{A}{\rho}\right) = 0 \quad (16)$$

which integrates to

$$\rho = \left[A + \frac{C(r)}{X^2 R^4}\right]^{\frac{1}{2}} \quad (17)$$

This becomes via equation(9)

$$\rho = \left[A + \frac{C(r)}{R'^2 R^4} \right]^{\frac{1}{2}} \quad (18)$$

Plugging in the expression of ρ from equation (10) we finally get

$$\frac{\dot{R}^2}{R^2} + 2 \frac{\dot{R}'}{R'} \frac{\dot{R}}{R} = \left[A + \frac{C(r)}{R'^2 R^4} \right]^{\frac{1}{2}} \quad (19)$$

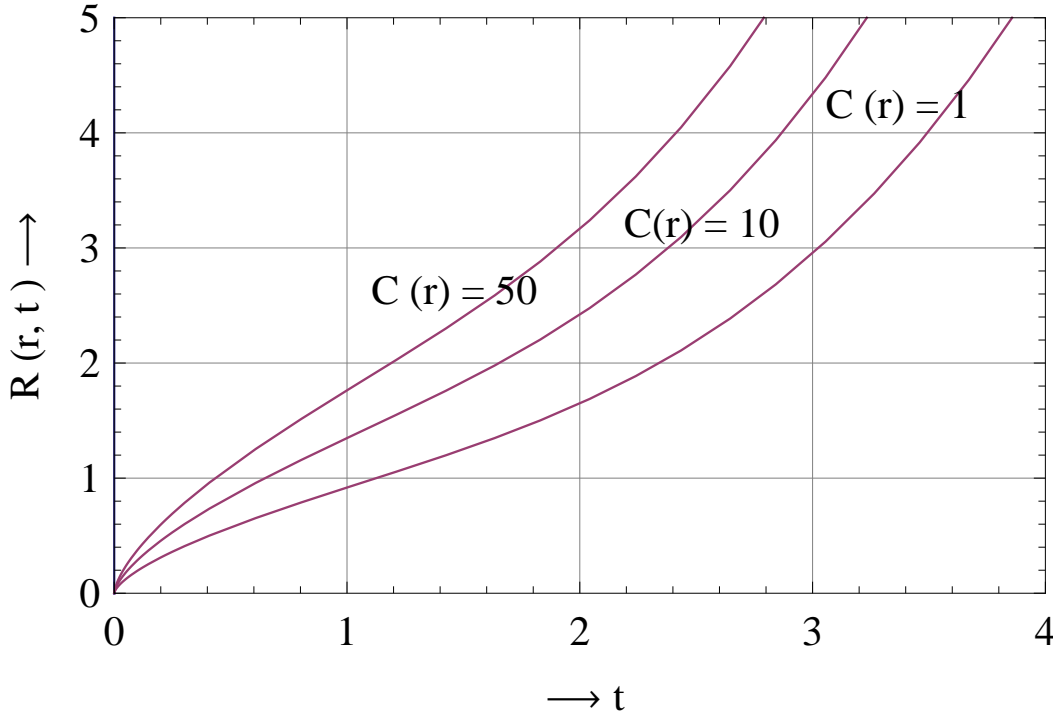


Figure 1: The variation of $R(r, t)$ and t for different values of $C(r)$ is shown. The graphs clearly show that acceleration increases for greater $C(r)$ i.e., greater inhomogeneity.

This is the key equation in most accelerating models dealing with a Chaplygin gas in homogeneous models except that $C(r)$ is now not a true constant but depends on space for inhomogeneous expansion. Moreover the above expression is not amenable to any closed form analytic solution but integration results in a hypergeometric series. Figure-1 shows that acceleration depends on $C(r)$, which represents the inhomogeneity. So $C(r)$ may be the measure of inhomogeneity in our case.

CASE A: At the early stage of the cosmological evolution when the scale factor $R(r, t)$ is relatively small the second term of the last equation (19) dominates and we get a sort of dust dominated universe for a particular value of $C(r) = \frac{16}{9}r^4$ yielding

$$R(r, t) = r [t + t_0(r)]^{\frac{2}{3}} \quad (20)$$

It has not also escaped our notice that with this scale factor we get a vanishing pressure when used in equation (11). Moreover for isotropic expansion ($X' = R$) we

get $\rho \sim \frac{1}{R^3}$ as in FRW universe. Relevant to point out that the expression (20) is not exactly Tolman-Bondi like and our line element reduces to

$$ds^2 = dt^2 - r^2 [t + t_0(r)]^{\frac{4}{3}} (dr^2 + r^2 d\Omega^2) \quad (21)$$

If, at this stage, we assume that $t_0(r)$ vanishes or losing its space dependence becomes a *true* constant (in that case a time translation is required) then we get

$$ds^2 = dt^2 - r^2 t^{\frac{4}{3}} (dr^2 + r^2 d\Omega^2) \quad (22)$$

This is a new solution and may be termed as the generalised Einstein- deSitter metric for the inhomogeneous spacetime. In the analogous homogeneous case for zero pressure dust with vanishing spatial curvature ($k = 0$) we get for FRW metric the wellknown Einstein-deSitter metric as ($R \sim t^{\frac{2}{3}}$). From equation (18) we get the expression of density as

$$\rho(r, t) \approx \frac{\sqrt{C}(r)}{R' R^2} = \frac{4}{3r [t + t_0(r)] \left[\frac{t+t_0(r)}{r} + \frac{2}{3} t'_0 \right]} \quad (23)$$

CASE B : ($R(r, t)$ is very large)

Type - 1:

In the late stage of evolution the second term of the RHS of the equation (19) vanishes and we get

$$\frac{\dot{R}^2}{R^2} + 2 \frac{\dot{R}'}{R'} \frac{\dot{R}}{R} = \sqrt{A} \quad (24)$$

This yields a solution

$$R(r, t) = R_0 \exp \left[\frac{A^{\frac{1}{4}}}{\sqrt{3}} (r + t) \right] \quad (25)$$

This is the wellknown de Sitter type of solution generalised to inhomogeneous space. $A^{\frac{1}{4}}$ behaves as Λ , the cosmological constant. However, if we make a radial coordinate transformation

$$\bar{r} = R_0 \exp \left[\frac{A^{\frac{1}{4}}}{\sqrt{3}} (r + t) \right] \quad (26)$$

the metric reduces to

$$ds^2 = dt^2 - \exp \left(2 \sqrt{\frac{A^{\frac{1}{2}}}{3}} t \right) d\bar{r}^2 - \bar{r}^2 \exp \left(2 \sqrt{\frac{A^{\frac{1}{2}}}{3}} t \right) d\Omega^2 \quad (27)$$

Type - 2: The volume expansion rate for our metric is defined through the 4-velocity of the fluid, u^a as

$$3H = u^a_{;a} = u_{a;b} g^{ab} = u_{a;b} h^{ab} \quad (28)$$

where

$$h^{ab} = g^{ab} + u^a u^b \quad (29)$$

As commented earlier in the discussion while the definition works perfectly well for a FRW like homogeneous distribution of matter it is always a bit ambiguous to define the deceleration parameter of an inhomogeneous anisotropic model because the relation (28) does not take into account the directional preference of the metric. For example, Tolman-Bondi has a preferred direction, being the radial one. We can still give an operational definition to the average *volume acceleration* of our model. For inhomogeneous model the directional preference need to be emphasized in the expression for expansion. We define a projection tensor t^{ab} that projects every quantity perpendicularly to the preferred spacelike direction s^a (and of course the timelike vector field, u^a) such that

$$t^{ab} = g^{ab} + u^a u^b - s^a s^b = h^{ab} - s^a s^b \quad (30)$$

For our metric (1),

$$p^a = \frac{\sqrt{1+f(r)}}{R'} \nabla \quad (31)$$

and the tensor projects every physical quantity in a direction \perp to s^a . One can now define the invariant expansion rates as

$$H_r = u_{a;b} s^a s^b = \frac{\dot{R}'}{R'} \quad (32)$$

$$H_{\perp} = \frac{1}{n} u_{a;b} t^{ab} = \frac{\dot{R}}{R} \quad (33)$$

so that

$$H = \frac{2}{3} H_{\perp} + \frac{1}{3} H_r \quad (34)$$

Evidently the above definition gives a sort of averaging over the various directions for our anisotropic model.

If one relaxes the condition of any particular preferred direction (like the radial one as in LTB model) one can explore the definition of the Hubble parameter in a more transparent way considering its directional dependence [31] as follows:

$$H = \frac{1}{3} u_a^a + \sigma_{ab} J^a J^b \quad (35)$$

where σ_{ab} is the shear tensor and J^a a unit vector pointing in the direction of observation. For an observer located away from the centre of the configuration it gives for our LTB case

$$H = \frac{\dot{R}}{R} + \left(\frac{\dot{R}'}{R} - \frac{\dot{R}}{R} \right) \cos^2 \theta \quad (36)$$

where θ is the angle between the radial direction through the observer and the direction of observation. Naturally when the two directions coincide, $\theta = 0$ we get

$H = H_r$ and for $\theta = \frac{\pi}{2}$ it is $H = H_\theta$. A definition of deceleration parameter in a preferred direction can also be given in terms of the expansion of the Luminosity distance D_L in powers of redshift of the incoming photons. For small z one gets

$$q = -\dot{H} \frac{d^2 D_L}{dz^2} + 1 \quad (37)$$

For $\theta = 0$ and $\theta = \pi/2$ the acceleration is respectively

$$q_r = -\left(\frac{R}{\dot{R}'}\right)^2 \left[\frac{\ddot{R}'}{R} - \frac{\sqrt{1+f}}{\dot{R}'} \left(\frac{\dot{R}'}{R}\right)' \right] \quad (38)$$

$$q_\perp = -\left(\frac{R}{\dot{R}}\right)^2 \frac{\ddot{R}}{R} \quad (39)$$

We shall subsequently see in section 3 that deceleration parameter defined this way has an important difference from what we later get in equation (56). Here the parameters do not depend solely on local quantities as opposed to the acceleration parameter of (56). For example we get via field equation(10-11)

$$q_\perp = \frac{M(r)}{R^3} \frac{1}{H_\perp^2} \quad (40)$$

where $M(r)$ is the mass of the fluid distribution upto the comoving radial coordinator. Thus the equation (40) tells us that here the deceleration parameter q_\perp depends on the total mass function and not on the local energy density of (48).

One can look into the above expression of q_\perp from a different standpoint also to assume a particular form of deceleration parameter as

$$q_\perp = -\frac{1}{H_\perp^2} \frac{\ddot{R}}{\dot{R}} = \frac{a - R^m}{b + R^m} \quad (41)$$

where a, b and m are constants. Straight forward integration of equation (41) yields

$$R(r, t) = R_0 \sinh^n \omega(r + t) \quad (42)$$

where, $n = \frac{2}{m}$, $a = (R_0)^{\frac{2}{n}} \left(\frac{1}{n} - 1\right)$ and $b = (R_0)^{\frac{2}{n}}$ such that we get from equation (41)

$$q_\perp = \frac{1 - n \cosh^2 \omega(r + t)}{n \cosh^2 \omega(r + t)} \quad (43)$$

showing that the exponent n determines the evolution of q_\perp . A little inspection shows that (i) $a < 0$, i.e., $n > 1$ gives acceleration, (ii) $a > 0$, i.e., $0 < n < 1$ gives the desirable feature of *flip*, although it is not obvious from our analysis at what value of redshift this *flip* occurs.

For $n = \frac{2}{3}$, equation (24) is satisfied for $A = \frac{16}{\sqrt{9}} \omega^4$ and in this case

$$q_\perp = \frac{3}{2} \text{sech}^2 \omega(r + t) - 1 \quad (44)$$

Figure -2 shows the variation of q_{\perp} and t for different values of r . We have seen from the graph that *flip* (t_c) occurs early at greater value of r , *i.e.*, acceleration depends on inhomogeneity. The *flip* time (t_c) will be in this case

$$t_c = \frac{1}{\omega} \left[-r + \text{sech}^{-1} \left(\sqrt{\frac{2}{3}} \right) \right] \quad (45)$$

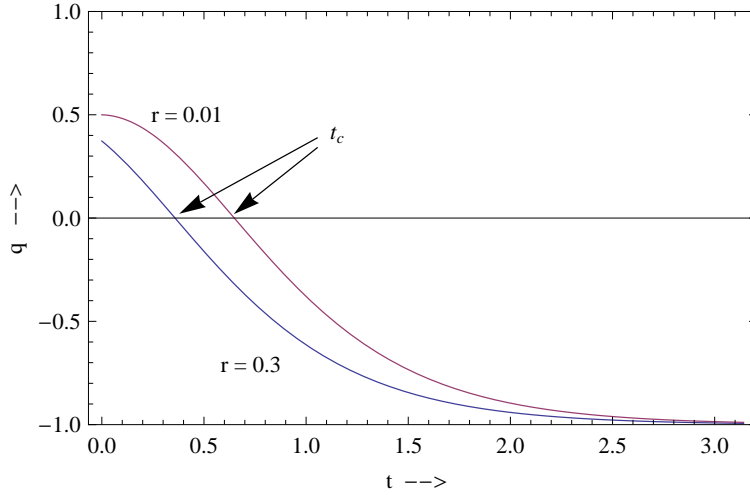


Figure 2: The variation of q_{\perp} and t for different values of r is shown. The graphs clearly show that *flip* (t_c) occurs early at greater value of r , *i.e.*, acceleration increases for greater r *i.e.*, greater inhomogeneity.

Type-3:

It also follows from equation (18) that for the late universe ($R \sim \infty$)

$$\rho \simeq \sqrt{A} + \frac{C(r)}{\sqrt{4A}} \frac{1}{R'^2 R^4} \quad (46)$$

$$p \simeq -\sqrt{A} + \frac{C(r)}{\sqrt{4A}} \frac{1}{R'^2 R^4} \quad (47)$$

This is a mixture of a Cosmological Constant \sqrt{A} with a type of matter obeying a ‘stiff fluid’ equation of state. However it should be pointed out that it is an inhomogeneous and anisotropic generalization of the well known FLRW situation characterized by $X(r, t) = R(r, t)$ where the quantities depend on time only.

Again as $R(r, t) \rightarrow \infty$, we asymptotically get $p = -\rho$ from this Chaplygin type of gas, which corresponds to an empty universe with Cosmological constant $\sqrt{\frac{A}{3}}$.

CASE C :

Now we are trying to solve the equation (19) using the method of separation of variables. Let $R(r, t) = g(r)a(t)$. From equation (19) we get

$$3 \frac{\dot{a}^2}{a^2} = \left(A + \frac{B}{a^6} \right)^{\frac{1}{2}} \quad (48)$$

where $B = \frac{C(r)}{g'^2 g^4} = \text{Constant}$ (say).

The equation (48) gives the hypergeometric solutions of $a(t)$ with t . The solution and other features are same like homogenous case [25, 32] such as

i) When $A = 0$, we get pressureless equation of state. Our solution reduces to FRW type and in this case $a(t) \sim t^{\frac{2}{3}}$.

ii) At early stage of evolution, i.e., for small value of $a(t)$, the equation (48) reduces to $3\frac{\dot{a}^2}{a^2} = \frac{\sqrt{B}}{a^3}$ and we get $a(t) \sim t^{\frac{2}{3}}$.

iii) At the late stage of evolution, i.e., $a(t)$ is large in this case, the equation (48) becomes (neglecting higher order terms)

$$3\frac{\dot{a}^2}{a^2} = \sqrt{A} + \frac{B}{\sqrt{4A}}a^{-6} \quad (49)$$

Solving the equation (49) we get the solution,

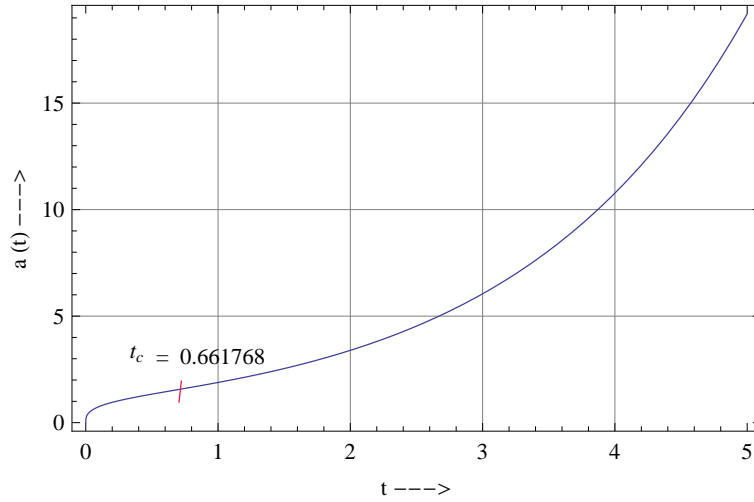


Figure 3: The variation of $a(t)$ vs t is shown in this figure. Taking $A=1$ & $B=12$. Flip occurs at $t = t_c = 0.661768$

$$a(t) = \sqrt{2} \frac{3^{\frac{1}{6}}}{\sqrt{A^{\frac{1}{6}}}} e^{-\frac{A^{\frac{1}{4}}}{\sqrt{3}}t} \left[e^{2\sqrt{3}A^{\frac{1}{4}}t} - \frac{B}{12\sqrt{A}} \right]^{\frac{1}{3}} \quad (50)$$

and also

$$g(r)^3 = \pm \frac{3}{\sqrt{B}} \int C(r) dr \quad (51)$$

So

$$R(r, t) = \sqrt{2} \frac{3^{\frac{1}{6}}}{\sqrt{A^{\frac{1}{6}}}} e^{-\frac{A^{\frac{1}{4}}}{\sqrt{3}}t} \left[\left(e^{2\sqrt{3}A^{\frac{1}{4}}t} - \frac{B}{12\sqrt{A}} \right) \frac{3}{\sqrt{B}} \int C(r) dr \right]^{\frac{1}{3}} \quad (52)$$

The nature of $R(r, t)$ with t for a typical r is shown in the fig. - 3.

In this case, $R(r, t) = 0$ at $t_0 = \frac{1}{2\sqrt{3}} A^{-\frac{1}{4}} \ln \left(\frac{B}{12\sqrt{A}} \right)$. Here $\ddot{R} = 0$ at $t = \frac{\ln \left[\sqrt{\frac{5B}{12\sqrt{A}}} + \frac{B}{\sqrt{6\sqrt{A}}} \right]}{\sqrt{3}A^{1/4}} = t_c$, which is the flip time.

CASE D : (For Negative Integration Constant)

Considering negative value of $C(r)$ presents interesting possibilities [33] since in that case the energy density increases with scale factor mimicing the phantom dark energy model and finally ending up as a cosmological constant. We get from equation(18)that for well behaved matter field the condition $R'^2 R^4 > \frac{C(r)}{A}$, *i.e.* need to be satisfied. So a minimal value of scale factor exists for a typical value r , which is $R(r, t)_{min} = \left[\frac{C(r)}{A} \right]^{\frac{1}{6}}$, pointing to a bouncing universe at early times. We thus see that the Chaplygin gas model interpolates between dust at small R and a cosmological constant at large R , but choosing a negative value for $C(r)$, this quartessence idea lose. Following Barrow [34] if we reformulate the dynamics with a scalar filed ζ and a potential V to mimic the Chaplygin cosmology, we see that a negative value for B dictates that we transform $\zeta = i\Psi$. In this case the expressions for the energy density and the pressure corresponding to the scalar field show that it represents a a phantom field. This implies that one can generate phantom-like equation of state from an interacting generalized Chaplygin gas dark energy model in LTB universe. This feature has been discussed in the past [12]in the context of an effective description of inhomogeneous model evolving like a homogeneous solution following an averaging technique and also in details in [35, 27] . In our case the bounce is inhomogeneous in the sense that each shell characterised by a constant radial coordinater bounces at its own time. So the bounce is not synchronous each shell sharing a local dynamics.

3 Raychaudhuri Equation :

It may not be out of place to address the situation discussed in the last section with the help of the well known Ray Chaudhuri equation [21], which in general holds for any cosmological solution based on Einstein's gravitational field equations. The Ray Chaudhuri equation is

$$\theta_{;\mu} v^\mu = \dot{v}^\mu_{;\mu} - 2(\sigma^2 - \omega^2) - \frac{1}{3} \theta^2 + R_{\nu\alpha} v^\nu v^\alpha \quad (53)$$

where the terms have their usual significance. With matter field expressed in terms of mass density and pressure Ray Chaudhury equation is finally given by,

$$\dot{\theta} = -2(\sigma^2 - \omega^2) - \frac{1}{3}\theta^2 - \frac{8\pi G}{2}(\rho + 3p) \quad (54)$$

in a co moving reference frame. Here p is the isotropic pressure.

With the help of equation(34) we get an expression for effective deceleration parameter as

$$q = -\frac{\dot{H} + H^2}{H^2} = -1 - 3 \frac{\dot{\theta}}{\theta^2} \quad (55)$$

which allows us to write,

$$\theta^2 q = 6\sigma^2 + 12\pi G(\rho + 3p) \quad (56)$$

With the help of the equations (15), (18) & (56) we finally get,

$$\theta^2 q = 6\sigma^2 + 12\pi G \left[-2A + \frac{C(r)}{R'^2 R^4} \right] \left[A + \frac{C(r)}{R'^2 R^4} \right]^{-\frac{1}{2}} \quad (57)$$

In our case the shear scalar evolves as

$$\sigma^2 = \sigma_{\mu\nu} \sigma^{\mu\nu} = \frac{2}{3} \left(\frac{\dot{R}'}{R'} - \frac{\dot{R}}{R} \right)^2 \quad (58)$$

CASE A : Early Stage: At the early phase of this evolution when the scale factor $R(r, t)$ is small enough the above equation reduces to

$$\theta^2 q = 6\sigma^2 + 12\pi G \frac{[C(r)]^{\frac{1}{2}}}{R' R^2} \quad (59)$$

It follows from the equation (59) that q , the deceleration factor is always positive. So accelerated expansion is absent in this dust dominated phase though inhomogeneity is present here. Interestingly this result is very similar to the work of Alnes *et al* [36].

CASE B : Late Stage :

Type - I: If we consider the late stage of evolution i.e., $R(r, t)$ is large enough in this phase, the second term of the RHS of the equation (19) vanishes and we get from equation (57),

$$\theta^2 q = 6\sigma^2 - 24\pi G \sqrt{A} \quad (60)$$

At this stage if we consider the scale factor given by equation (42) ($n = \frac{2}{3}$) the shear scalar becomes $\sigma^2 = \frac{8}{3}\omega^2 \text{cosech}^2 [2\omega (r + t)]$ & $A = \frac{16}{9}\omega^4$. The equation (57) reduces to

$$\theta^2 q = 16\omega^2 \text{cosech}^2 [2\omega (r + t)] - 32\pi G \omega^2 \quad (61)$$

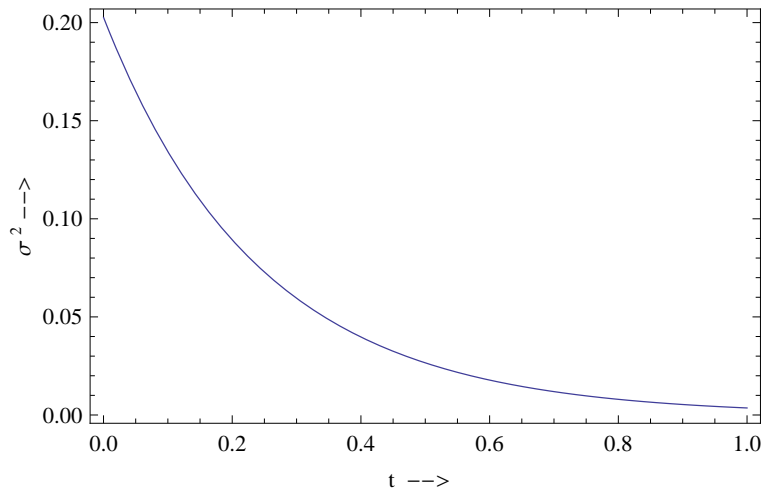


Figure 4: The variation of σ^2 vs t is shown in this figure. Taking $\omega = 1$ & $r = 1$.

In figure - 4 shows σ^2 vs t for a particular r . In this graph we have seen that as t increases σ^2 decreases, *i.e.*, when $t \rightarrow \infty$, $\sigma^2 \rightarrow 0$. So initially we get the decelerating universe and after *flip* it becomes accelerating in line with current observational result (see equation (60)).

Type - II: Again if we consider first order approximation of equation (57), neglecting higher order terms, we get

$$\theta^2 q = 6\sigma^2 + 24\pi G A \left[-A + \frac{C(r)}{R^2 R^4} \right] \quad (62)$$

Let $R(r, t) = g(r)a(t)$, so in this case $\sigma = 0$ which follows from the equation (58). Now the equation (62) reduces to

$$\theta^2 q = 24\pi G A \left[-A + \frac{B}{a^6} \right] \quad (63)$$

It follows from the equation (63) that flip occurs when $a(t) = \left(\frac{B}{A}\right)^{\frac{1}{6}}$. Now $q < 0$, at $a(t) > \left(\frac{B}{A}\right)^{\frac{1}{6}}$ *i.e.*, acceleration takes place in this case.

Negative Constant : If $C(r) < 0$, then $B < 0$, the equation (63) then becomes

$$\theta^2 q = -24\pi G A \left[A + \frac{B}{a^6} \right] \quad (64)$$

From the above equation we have seen that always $q < 0$, which means we get always accelerating universe.

4 Concluding Remarks

The present work may be looked upon as an extension of one of our recent publications where we examined the possibility in a higher dimensional LTB model if the inclusion of extra space jointly with inhomogeneity can induce late inflation in a dust model. While total volume acceleration is ruled out we found that preferential acceleration in radial direction is possible if the angular direction decelerates fast enough or *vice versa*. Here we have taken a Chaplygin type of gas as matter field to work out the same problem in a 4D spacetime. While there is a proliferation of work in the literature on homogeneous FRW model with Chaplygin gas we have not much come across work of similar type in inhomogeneous spacetime. Given the fact that it is difficult, if not a little confusing to define uniquely a deceleration parameter in inhomogeneous, anisotropic model we have nevertheless got the following definitive results.

1. Aside from space dependence the mathematical structure and its followup in the section 2 is essentially similar to the works of homogeneous spacetime except the appearance of the term, $C(r)$ in equation(17), which unlike its homogeneous counterpart is not a true constant but depends on the space coordinate. Its presence introduces all the differences in cosmic evolution. Like FRW models our field equations are amenable to exact solution only at extreme values. We find that at

early stage our solution reduces to inhomogeneous analogue of the Einstein-deSitter type of solution.

2. In line with current observational findings our model accounts for early deceleration and late acceleration showing the desirable phenomenon of *flip* in all the examples we examined. But as expected here the time of flip depends on space coordinate also. So flip is not synchronous as in FRW cases, occurring at different shells at different instants. So flip here is local, not global. Moreover flip occurs early for larger ' r '. So for our spherically symmetric model the outer shells will start acceleration earlier and this is also a good news *vis a vis* when posited against the problem of shell crossing singularity *generically* associated with inhomogeneous models.

3. Another interesting situation discussed is the possibility of bounce of our model from a minimum when the arbitrary function of integration, $C(R)$ assumes a negative value. The bounce also shares the inhomogeneous characteristic of our model, the different shells characterised by r -constant hypersurfaces bounce at different instants. Moreover we have here taken the original Chaplygin gas in our analysis but now generalised type of equation of state [37] are being increasingly used with greater freedom. In our future work we try to extend this work with these modified Chaplygin gas equations.

4. To end the section a final remark may be reemphasised regarding the apparent accelerated expansion of the universe. To explain the SNIa observations the concept of accelerated expansion of the universe need to be invoked only for a FRW type of model. But one should point out that the Luminosity distance- Redshift relation, not the accelerated expansion is the quantity that can be directly measured. And within inhomogeneous models one gets better fit without the need to introduce the local accelerated expansion and consequent hypothesis of any extraneous, unphysical matter field with large negative pressure.

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